# Finite Element – A Way to Solve for Scale Problems

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## 1 Introduction

The purpose of using a finite element code is to improve the relation between point information, e.g. transmissivities based on pumping tests, and spatial distributed parameter values that are typically used in numerical models. Other purposes are to better describe the exchange of groundwater and surface water in areas near rivers and to use a finer discretization near abstraction wells.

A small literature study was carried out on existing finite element groundwater codes:

| | |
|---|---|
| MicroFem | Confined, semi-confined, phreatic, stratified and leaky multi-aquifer systems can be simulated with a maximum of 20 aquifers. www.microfem.com |
| Feflow | Density-dependent flow with mass and heat transport, saturated/unsaturated zone, confined and unconfined aquifers, and multiple free surfaces for perched water tables. www.scisoftware.com |
| FemWater | Saturated/unsaturated, density dependent, flow and transport. Modules for simulating evaporation/infiltration/seepage on the soil-air interface and adsorption/dispersion/first-order decay. |
| Aqua3D | Solves transient groundwater flow with heterogeneous and anisotropic flow conditions. Also solves transient transport of contaminants and heat with convection, decay, adsorption, and velocity-dependent dispersion. www.scisoftware.com |
| 3DFemfat | Saturated/unsaturated flow and transport, density-dependent flow and transport. Ideal for applications to large field problems. www.scisoftware.com |
| Watflow/3D | Heterogeneous confined/unconfined aquifer systems, saturated/unsaturated conditions. Deformable elements for the free surface. Includes modules for parameter sensitivity and automatic calibration. [*Beckers et al.*, 2000]. |

The above mentioned codes are groundwater codes with little focus on the interaction between groundwater and surface water. One of the areas of special interest is this interaction and its scale dependency. The interaction between a groundwater model that operates in a 1000 m grid scale and a river model that operates on a 1 m scale is rather difficult to describe: the topography of the river valley may not be described correctly, the groundwater table varies often close to streams, evaporation from the wetland close to the river may not be described correctly on this scale etc. Therefore, it was decided to try to develop a finite element (FE) groundwater model for MIKE SHE and couple this model with a river model. The FE grid should be able to describe the variations close to the river at an appropriate scale and thereby improve the model reliability.

A FE groundwater model has been developed and tested against analytical solutions for pure groundwater problems. However, it was decided to stop this development and not couple the groundwater model with a river model due to different circumstances: development costs were very high, the prospects for a successful finalisation of the development were very bad and GEUS would not be able to test and apply it during the construction of the National Water Resources model.

In the following the development of the FE code is described together with the tests carried out to validate the code.

# 2 Programming of the Finite Element Code

## 2.1 Model Structure

The finite element code is written in Fortran90 and is a part of a module-based system. The groundwater module uses the same solver(s), which can be used by other modules (e.g., hydrodynamic, advection/dispersion, transport modules etc.). The core development of engines was financed under internal development fund and focused on 2D and 3D models for flow and transport in open water.

## 2.2 Solution Method and Solver

The governing equations are discretized using the standard Galerkin finite element method. The finite element code solves the transient, saturated flow equation in both 2D and 3D; in equation (2.1) shown for a 3D flow in a confined aquifer [*Istok*, *1989*]:

$$\frac{\partial}{\partial x}\left(K_X \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)=S_S \frac{\partial h}{\partial t} \quad (2.1)$$

where $K_i$ is the hydraulic conductivity [L/T] in Cartesian co-ordinates (i = x, y, and z), h is the potential head [L], $S_S$ is the storage term [$L^{-1}$], and t is time [T].

The 2D Boussinesq's equation for transient flow in phreatic aquifer reads:

$$\frac{\partial}{\partial x}\left(K_X h \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y h \frac{\partial h}{\partial y}\right)=Sy \frac{\partial h}{\partial t} \quad (2.2)$$

where Sy is the specific yield [-].

Five solution techniques are available to solve the differential algebraic equations (DAE's):

1: Three-step explicit method
2: Adam-Mouton predictor/corrector method
3: Implicit trapezoid rule
4: Implicit BDF method (transient)
5: Newton-Raphson method (quasi-stationary solution)
Only method 4 and 5 has been used for the groundwater module.

The linear equation system (LES) can be solved using two different methods:

1: Gauss elimination using sparse technique
2: GMRES with preconditioning using incomplete Gauss elimination, SPGMR (scaled preconditioned generalised minimum residual method)
Only method 2 has been used for the groundwater module.

It is a 3D finite element code, which solves the transient, saturated flow equation for a fully unstructured three-dimensional grid of elements. For practical purposes, however, the model setup will in most cases be a layered model, simply because a totally unstructured 3D grid would be very difficult to visualise and to make geological interpretations.

## 2.3 Properties of the Groundwater Module

The code handles both confined and unconfined conditions, and hence, also two types of storage terms are implemented: the specific yield for unconfined conditions and the storage coefficient for confined conditions. The storage terms can be constant or variable in space, and are of course only relevant for the transient test cases. In this present version the storage terms are assumed not to vary in time, which is thought to be a reasonable assumption.

The code handles unsaturated/saturated conditions for all layers. For unsaturated conditions the hydraulic conductivity decreases as a function of the (negative) pressure head p above the groundwater table [*Beckers et al.*, 2000]:

$$S(p) = \begin{cases} S_r + (1-S_r)e^{\varepsilon p} & p < 0 \\ 1 & p \geq 0 \end{cases} \qquad (2.3)$$

where S is an artificial 'saturation', $S_r$ is a residual 'saturation', and ε is a parameter controlling the exponential drop-off from full to residual 'saturation'.

A variable sink/source term in time and space can be added to any element in both 2D and 3D.

The hydraulic conductivities Kx, Ky, and Kz can be either constant or vary in space.

Groundwater flow is calculated indirectly from Darcy's law, which is a little less accurate than calculating the flow directly from the governing equations. On the other hand, it is cheaper to do the flow calculations this way in terms of CPU time.

Compared to the present groundwater module of MIKE SHE, the finite element code handles most of the basic features, except from the drainage function:

MIKE SHE - SZ FEM - GroundWater

F.1.1 Grid codes

Topography ………………………. [grid] (unstructured grid)

F.1.2.a Lower levels

F.1.1 Precipitation distribution

Precipitation time series …………… [source] (t, x, y)

F.1.2.b Hydrological parameters

Kx, Ky, Kz, Sfree, Sart ………….. [conductivity] and [storage] (x, y, z)

F.1.4 Initial head ………………………… [starting_conditions] (x, y, z)

F.1.5 Boundary conditions ………………. [boundary_conditions] (head or flux BC)

Abstractions ………………………. [source_and_sink] (t, x, y, z)

F.1.6 Drainage …………………………… Not implemented

# 3 Documentation – Analytical Solutions

The best way to test a numerical code is probably to verify the results with an analytical solution. If this is not possible, the second best is to verify the results with an existing, reliable numerical code.

In this project three test examples were carried out: 1) a circular island with recharge and a pumping well in the centre, 2) a phreatic surface (unsaturated elements), and 3) variation in hydraulic conductivity. The two first test examples can be compared to analytical solutions, but no analytical solution exist for the third test example.

## 3.1 Analytical Solution for a Circular Island

The drawdown is calculated for a circular island with a constant abstraction rate in a well in the centre of the island, Figure 3.1. A constant recharge is added on top of the aquifer. Two different examples can be evaluated: steady-state and transient, however, only the analytical solution at steady-states will be evaluated here.

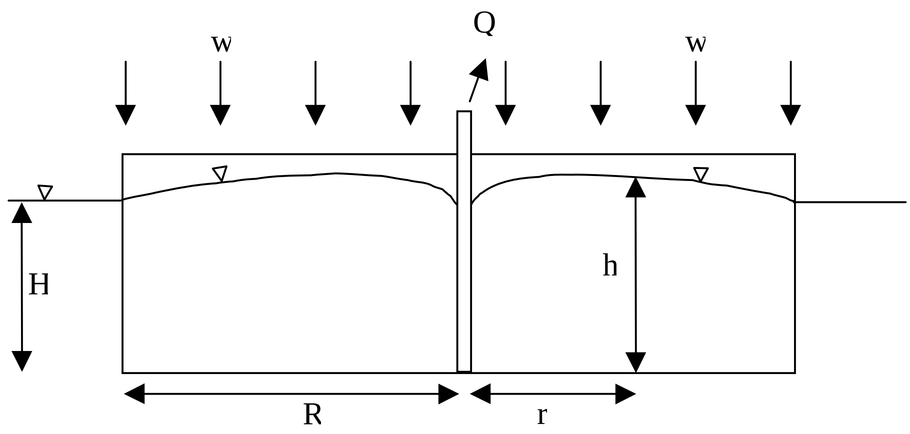


*Figure 3.1 The circular island test example.*

The steady-state solution of the hydraulic head, h [L], as a function of the distance from the well, r [L], is given by:

$$H^2 - h^2 = \frac{Q_w}{\pi K}\ln\frac{R}{r} - \frac{w}{2K}\left(R^2 - r^2\right),\ 0 < r \le R \qquad (3.1)$$

H Initial saturated thickness [L]
R Radius of the island [L]
$Q_w$ constant flow rate abstracted from the well [$L^3/T$]
K Hydraulic conductivity [L/T]
w Recharge [L/T]

In order to compare the finite element and MIKE SHE codes to the analytical solution the maximum of the hydraulic head is calculated. The maximum hydraulic head is found by differentiate equation (3.1) with respect to r and set the derivative equal to zero. By doing so, one finds that h has a maximum if

$$\frac{dh}{dr} = 0 \Rightarrow r = \sqrt{\frac{Q_w}{\pi w}} \qquad (3.2)$$

In the case of no abstraction ($Q_w = 0$) r = 0, and from equation (3.1) one yields:

$$h_{max} = \sqrt{H^2 + \frac{wR^2}{2K}} \qquad (3.3)$$

In the case of an abstraction rate $Q_w \neq 0$, the maximum hydraulic head is found from equation (3.1) at the radius calculated from (3.2).

It is not just the maximum hydraulic head, which has to match; the head as a function of the distance r = 0 to r = R can be plotted to compare the analytical solution with the numerical models (finite element and MIKE SHE).

## 3.2 Analytical Solution for a Phreatic Surface

The position of the phreatic surface in an aquifer with fixed head boundary conditions in both ends is illustrated in Figure 3.2. Two different examples can be evaluated: steady-state and transient, however, only the analytical solution at steady-state will be evaluated here. The transient case occurs when the initial head is at a level $H_2$, and for t > 0 drops to $H_1$ at x = 0. Please notice that Figure 3.2 is not to scale! The reason for this test example is to model partly saturated cells/elements and test the decreasing water content of the cells/elements in the transient case.

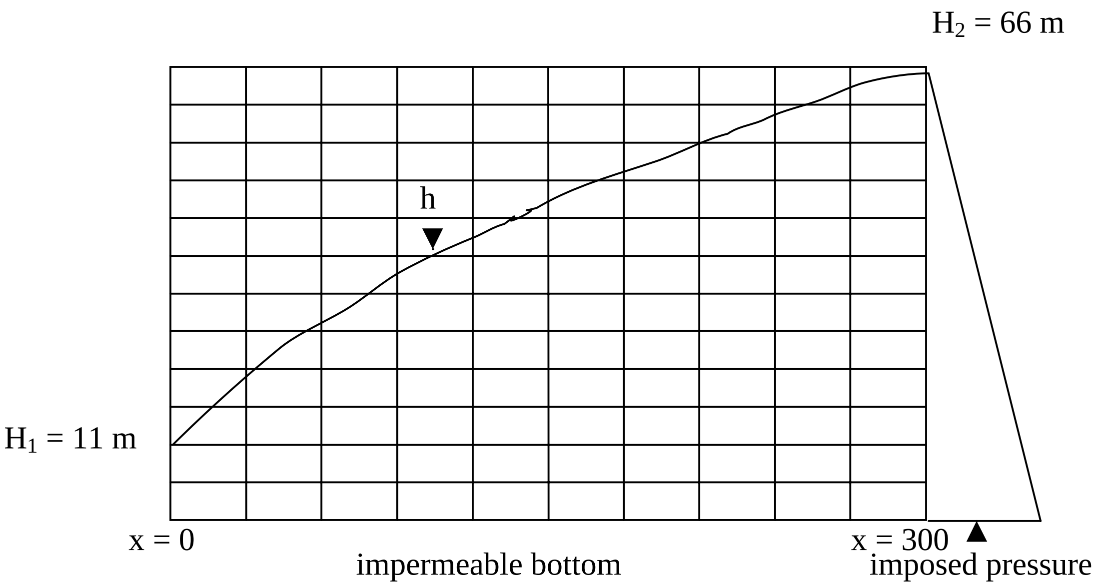


*Figure 3.2 The phreatic surface test example – not to scale.*

The analytical solution of the hydraulic head at steady-state is given by:

$$\frac{d^2(h^2)}{dx^2} = 0 \Leftrightarrow h^2 = C_1 x + C_2 \Rightarrow h(x) = \sqrt{14.1167x + 121}, x = [0;300] \qquad (3.4)$$

where the constants $C_1$ and $C_2$ are determined from the boundary conditions:

$H_1(x = 0) = 11$ m and $H_2(x = 300) = 66$ m

Equation (3.4) is actually only valid if two assumptions (due to Dupuit) are made:

1. dh/dl can be represented by dh/dx, where dl is the length of the curve
2. all flow lines in the aquifer are horizontal and equipotential lines vertical

At x = 300 m the assumptions are reasonable, however, at x = 0 the flow lines are obviously not horizontal. But even when the assumptions are not completely fulfilled, the finite element and MIKE SHE results are close to the analytical solution, see Chapter 4.2.

# 4 Test Examples

The intention was to test the finite element code for a few simple test cases and compare with MIKE SHE and analytical solutions if possible. Three test examples were carried out: 1) a circular island with recharge and a pumping well in the middle, 2) a phreatic surface (unsaturated elements), and 3) variation in hydraulic conductivity. An overview of the various test examples are listed below:

**Circular island:**
- Analytical
- MIKE SHE
  - 2D (number of computational points: 8048)
    - Steady-state
    - Transient
  - 3D (number of computational points: 40240)
    - Steady-state
    - Transient
- FEM
  - 2D (elements: 1274, nodes: 677)
    - Steady-state
    - Transient
  - 3D (elements: 5096, nodes: 3385)
    - Steady-state
    - Transient

**Phreatic surface, 3D:**
- Analytical
- MIKE SHE (number of computational points: 348)
  - Steady-state
  - Transient
- FEM (elements: 480, nodes: 429)
  - Steady-state
  - Transient

**K-variation, 3D, (clay lens):**
- MIKE SHE (number of computational points: 3105)
  - Steady-state
  - Transient
- FEM (elements: 6250, nodes: 4056)
  - Steady-state
  - Transient

**Lille Å, 3D, transient, (real test case):**
- MIKE SHE (number of computational points: 5292)
- FEM (elements: 10204, nodes: 6805)

## 4.1 Circular Island

Parameter values:
Diameter 2R = 10,000 m, i.e., R = 5,000 m
Initial saturated aquifer thickness H = 8 m
Hydraulic conductivity K = $10^{-3}$ m/s
Storage Sy = 0.2 (unconfined), Ss = 0.0001 $m^{-1}$ (confined)
Recharge w = 0.01 mm/h = 0.0876 m/yr = $2.778 \times 10^{-9}$ m/s
Pumping (sink term) $Q_w$ = 1000,000 $m^3$/yr = 0.0317 $m^3$/s

Discretization:
MIKE SHE: DX = DY = 100 m, DZ = 2 m
FEM: DX = DY = 400 m (approx.), DZ = 2.5 m

Analytical solutions:
In the case of no abstraction ($Q_w$ = 0) r = 0, and from equation (3.1) one yields:

$$h_{max} = \sqrt{H^2 + \frac{wR^2}{2K}} = 9.9359 \text{ m.}$$

With an abstraction rate of $Q_w$ = 0.0317 $m^3$/s and a recharge of $2.778 \times 10^{-9}$ m/s, r ≈ 1906 m, and from equation (3.1) one yields $h_{max}$ = 9.1620 m.

*Table 4.1 Results and CPU times for the different scenarios.*

| Model | Dim. | Mode | Pump. | $H_{max}$ / [m] | CPU* / [s] |
|---|---|---|---|---|---|
| Analytical | - | - | - | 9.9359 | - |
| Analytical | - | - | + | 9.1620 | - |
| MIKE SHE | 2D | Steady-state | - | 9.9415 | ~ 5 |
| MIKE SHE | 2D | Steady-state | + | 9.1851 | ~ 5 |
| MIKE SHE | 2D | Transient | - | 9.9415 (40 years) | ~ 900 |
| MIKE SHE | 2D | Transient | + | 9.1851 (40 years) | ~ 1140 |
| MIKE SHE | 3D | Steady-state | - | 9.9416 | ~ 60 |
| MIKE SHE | 3D | Steady-state | + | 9.1851 | ~ 80 |
| MIKE SHE | 3D | Transient | - | 9.9414 (40 years) | ~ 4320 |
| MIKE SHE | 3D | Transient | + | 9.1850 (40 years) | ~ 4320 |
| FEM | 2D | Steady-state | - | 9.9359 | 0.15 |
| FEM | 2D | Steady-state | + | 9.1697 | 0.16 |
| FEM | 2D | Transient | - | 9.9359 (40 years) | 29 |
| FEM | 2D | Transient | + | 9.1697 (40 years) | 29 |
| FEM | 3D | Steady-state | - | 9.9232 | 11 |
| FEM | 3D | Steady-state | + | 9.1520 | 6 |
| FEM | 3D | Transient | - | 9.9232 (40 years) | 673 |
| FEM | 3D | Transient | + | 9.1520 (40 years) | 781 |

* See appendix A

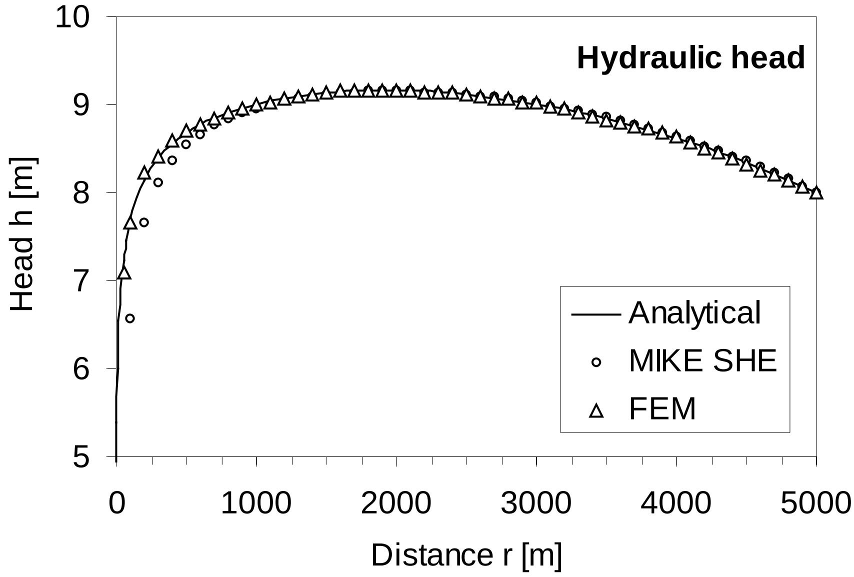


*Figure 4.1 Hydraulic head in a 2D steady-state scenario with a pumping well at r = 0 and constant head at r = 5000 m – not to scale!*

In Figure 4.1 the hydraulic head is compared for the analytical solution, MIKE SHE, and the finite element code. More than approximately 1000 m from the well, the three models are almost identical. Close to the well, however, some discrepancies occur due to the discretization of the cells/element.

The finite element model used two different triangular grids with a 'grid size' of approximately 100 and 400 m, respectively. The 'grid size' varies slightly because of the irregular grid, but on average the distance between the elements are 100 and 400 m, respectively. Therefore, it is not possible to extract the value at e.g. exactly r = 3000 m in the finite element model, but one has to use the closest element. Using the 100-m grid, as in Figure 4.1, the horizontal displacement error is at maximum 50 m, which is not very important for the hydraulic head far from the well. Close to the well the error will be more pronounced because of the gradient of the water table. The advantage of a finite element model is to refine the discretization close to e.g. a well, but this option has not been tested so far.

MIKE SHE has the same inaccuracies close to the well due to the discretization as the finite element showed, perhaps even worse, although the same discretization was used. This is because the MIKE SHE setup uses an even number of cells (100) in the X and Y direction, and hence, the cell closest to the centre is 50 m off, since the head is calculated in the middle of the cell. Using e.g. 101 cells instead would decrease the error close to the well, but then the analytical solution would also change because the radius would be R = 5050 m.

In Table 4.1 the maximum hydraulic head for various tests (2D/3D, with or without pumping, steady-state/transient) are compared to the analytically calculated head. The transient simulations run to t = 40 years in order to compare the transient results with the steady-state solution.

The finite element model gives the same results for steady-state and transient (after 40 years), but a minor discrepancy between 2D and 3D. The discrepancy is due to the different ways the phreatic surface is calculated in 2D and 3D. In 2D the phreatic surface is calculated straightforward, while in 3D it is calculated using equation (2.3) in partly saturated elements, which will be the situation in the top layer. In the finite element model there is a calculation point in exactly (x,y) = (0,0) so for the 2D tests without pumping a perfect match of the analytical solution is achieved (9.9359 m). If pumping is included the results are a little off due to the horizontal displacement of the calculation point compared to the calculated distance of 1906 m from the centre of the island.

In MIKE SHE the results are nearly the same for both steady-state/transient and 2D/3D, however the results are a little off compared to the analytical results. This is probably due to the discretization as discussed above.

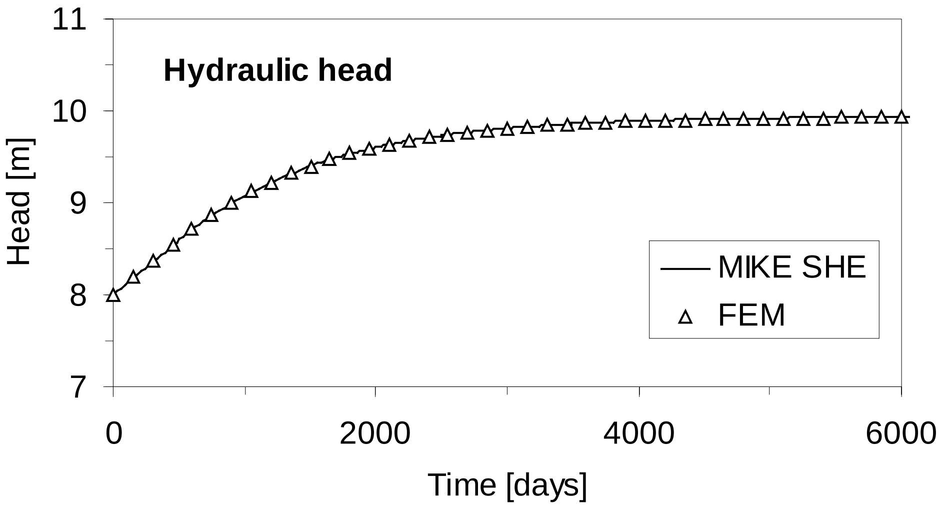


*Figure 4.2 Hydraulic head in the middle of the island in a 2D transient scenario without pumping.*

An example of a transient scenario is shown in Figure 4.2 where the hydraulic head is plotted as a function of time. The finite element model is almost identical to the MIKE SHE solution, and similar results were found for other scenarios including 3D and/or pumping.

## 4.2 Phreatic surface

Parameter values:
Dimension, MIKE SHE (X, Y, Z) = 300 m $\times$ 90 m $\times$ 66 m
Dimension, FEM (X, Y, Z) = 300 m $\times$ 60 m $\times$ 66 m
Initial saturated aquifer thickness H = 66 m
Hydraulic conductivity K = $1.5 \times 10^{-5}$ m/s
Storage Sy = 0.2 (unconfined), Ss = 0.0001 $m^{-1}$ (confined)
Phreatic surface parameters (FEM) Sr = 0.005, esp = 100.0

Discretization DX = DY = 30 m, DZ = 5.5 m (see Figure 3.2)

Boundary conditions:
X = 0: Fixed head at H = 11 m
X = 300 m: Fixed head at H = 66 m

Analytical solution:

$$h(x) = \sqrt{14.1167x + 121} \qquad , x = [0; 300 \text{ m}]$$

*Table 4.2 Calculated hydraulic head as a function of the distance x and CPU times.*

| Distance | Analytical | MIKE SHE Steady-state | MIKE SHE Transient | FEM Steady-state | FEM Transient |
|---|---|---|---|---|---|
| 0 | 11.0000 | 11.0000 | 11.0000 | 11.0000 | 11.0000 |
| 30 | 23.3345 | 24.0950 | 24.0950 | 23.0851 | 23.3931 |
| 60 | 31.1127 | 31.7269 | 31.7269 | 30.8942 | 31.1488 |
| 90 | 37.3028 | 37.7475 | 37.7477 | 36.9883 | 37.1784 |
| 120 | 42.6028 | 42.9078 | 42.9079 | 42.2464 | 42.3699 |
| 150 | 47.3128 | 47.5194 | 47.5195 | 46.9667 | 47.0499 |
| 180 | 51.5946 | 51.7304 | 51.7303 | 51.2826 | 51.3522 |
| 210 | 55.5473 | 55.6186 | 55.6185 | 55.2723 | 55.3469 |
| 240 | 59.2368 | 59.2467 | 59.2467 | 59.0057 | 59.0693 |
| 270 | 62.7097 | 62.6846 | 62.6846 | 62.5437 | 62.5779 |
| 300 | 66.0000 | 66.0000 | 66.0000 | 66.0000 | 66.0000 |
| CPU* / [s] | - | ~ 1 | ~ 30 | 1.44 | 4824 |

* See Appendix A

For the MIKE SHE model the transient simulation reaches steady-state (with four digits) after approximately 820 days (the simulation runs to 1000 days which takes about 30 s of CPU time). The simulation time of the steady-state solution depends on the accuracy criteria, and thereby number of iterations; an accurate solution can be found within a few seconds. Even though Dupuit's assumption (especially the restriction on horizontal flow lines) are not completely fulfilled, MIKE SHE performs well compared to the analytical solution, see Table 4.2 and Figure 4.3. Almost identical results were found with MIKE SHE for the steady-state and transient simulation.

The finite element model performs well compared to the analytical solution, too, with a simulation time of about 1-2 s of CPU time at steady-state. The transient simulation is also close to the analytical solution, however, the difference between the steady-state and transient simulation is much larger than for MIKE SHE and the computational time is more than an hour. On the other hand, the finite element model is closer to the analytical solution, especially the transient scenario. The difference is probably due to the fact that Dupuit's assumption is not completely fulfilled.

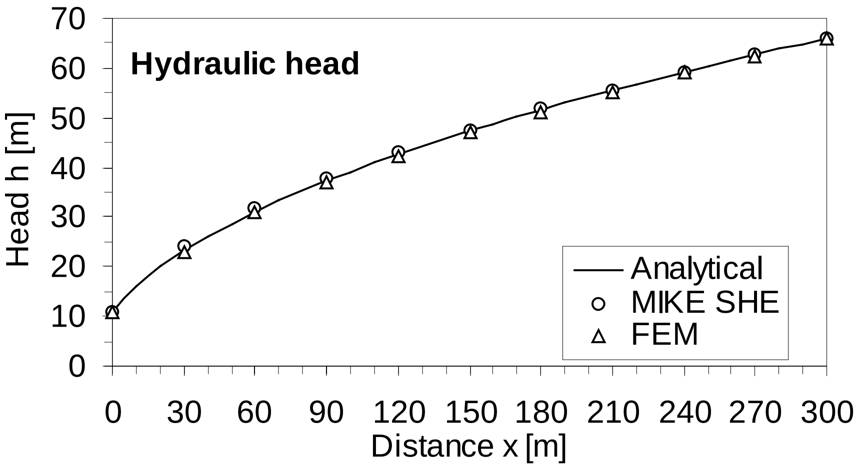


*Figure 4.3 Hydraulic head in a steady-state scenario – not to scale.*

In Figure 4.4 the hydraulic head is plotted as a function of time at a distance of x = 150 m and a depth of z = 11 m (corresponding to layer 11 in MIKE SHE). There is a discrepancy between the finite element model and MIKE SHE, especially in the beginning of the simulation. This is probably due to the choice of phreatic surface parameters from equation (2.3). The same parameters as for the circular island are used, but the conceptual model is more complex in this test case with several dry cells.

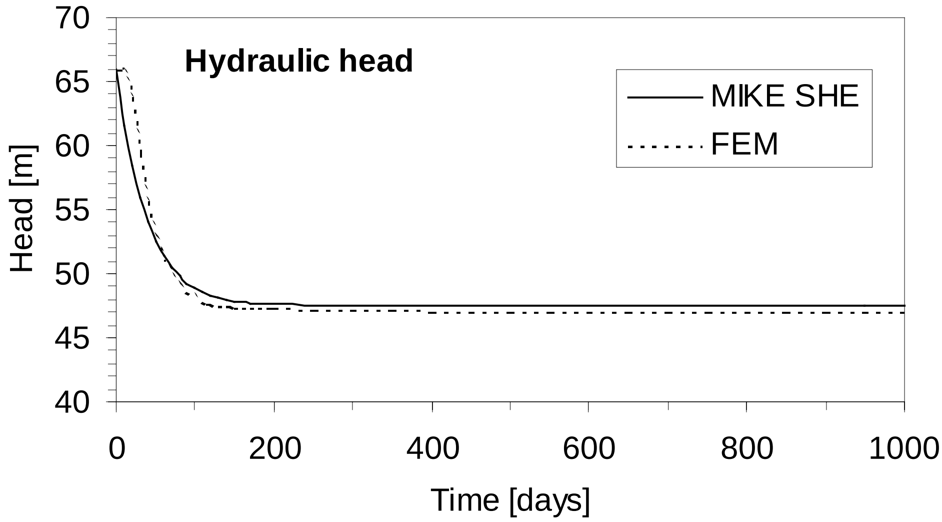


*Figure 4.4 Hydraulic head at x = 150 m (depth z = 11 m) in a transient scenario.*

## 4.3 K-variation (clay lens)

Parameter values:
Dimension (X, Y, Z) = 10000 m × 10000 m × 10 m
Initial saturated aquifer thickness H = 8 m
Hydraulic conductivity (sand) $K = 10^{-3}$ m/s
Hydraulic conductivity (clay lens) $K = 10^{-6}$ m/s
Recharge w = 0.01 mm/h = 0.0876 m/yr = $2.778 \times 10^{-9}$ m/s
Pumping (sink term) Q = 1000,000 $m^3$/yr = 0.0317 $m^3$/s
Well (MIKE SHE), (X, Y) = (5000, 5000), Layer 3 (-6 < Z < -4)

Well (FEM), (X, Y, Z) = (5000, 5000, -6)
Storage Sy = 0.2 (unconfined), Ss = 0.0001 $m^{-1}$ (confined)
Phreatic surface parameters (FEM) Sr = 0.005, esp = 100.0

Discretization:
DX = DY = 400 m, DZ = 2 m

Size and location of the clay lens:
0 < x < 4000 m and 0 < y < 10000 m and 4 < z < 6 m
(MIKE SHE: 10 × 25 cells in layer 3)

*Table 4.3 Results and CPU times for the different tests*

| Model | Mode | Lens | $H_{max}$ / [m] | CPU* / [s] |
|---|---|---|---|---|
| MIKE SHE | Steady-state | - | 9.3103 | ~ 5 |
| MIKE SHE | Steady-state | + | 9.5258 | ~ 5 |
| MIKE SHE | Transient | - | 9.3103 (40 years) | ~ 2700 |
| MIKE SHE | Transient | + | 9.5257 (40 years) | ~ 3000 |
| FEM | Steady-state | - | 9.5236 | 16 |
| FEM | Steady-state | + | 9.7197 | 17 |
| FEM | Transient | - | 9.5238 (40 years) | 1147 |
| FEM | Transient | + | 9.7197 (40 years) | 1156 |

* See Appendix A

In Figure 4.5 the hydraulic head is shown in a cross section in the middle of the ‘square island’ (y = 5000 m) for a steady-state scenario including the clay lens. There is a higher hydraulic head on the left side of the pumping well (x < 5000 m) because of the clay lens. The finite element model simulates a significant higher hydraulic head than MIKE SHE does. Close to the well it can be explained by the fact that MIKE SHE calculates the head in cells and the finite element code in nodal points, which is also seen for the circular island in Figure 4.1. Far from the well there is no good explanation to the discrepancy between the two models. It may be due to the phreatic surface parameters, but this has not been tested in detail. The reason why the MIKE SHE curve stops at x = 9600 m is because only 25 cells were used. (If an even number of cells had been used, it had not been possible to place the well exact in the middle of the catchment area).

Figure 4.6 shows the transient scenario in a cell/element where the maximum hydraulic head is calculated. Again the finite element model simulates a higher head than MIKE SHE does but in this scenario it is not guaranteed, that the maximum head is calculated at exact the same horizontal location because of the discretization. However, the overall trend of the rise in hydraulic head is well captured.

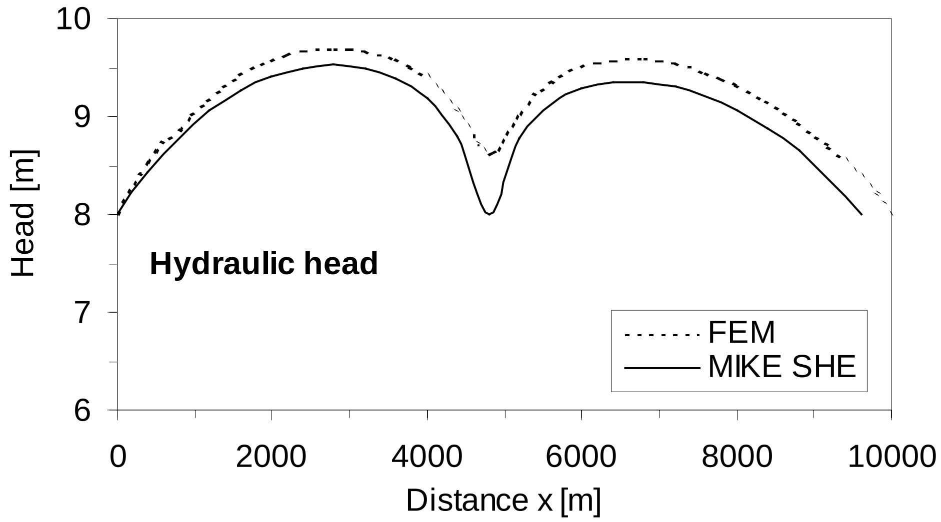


*Figure 4.5 Hydraulic head at a cross section y = 5000 m in a steady-state scenario – not to scale.*

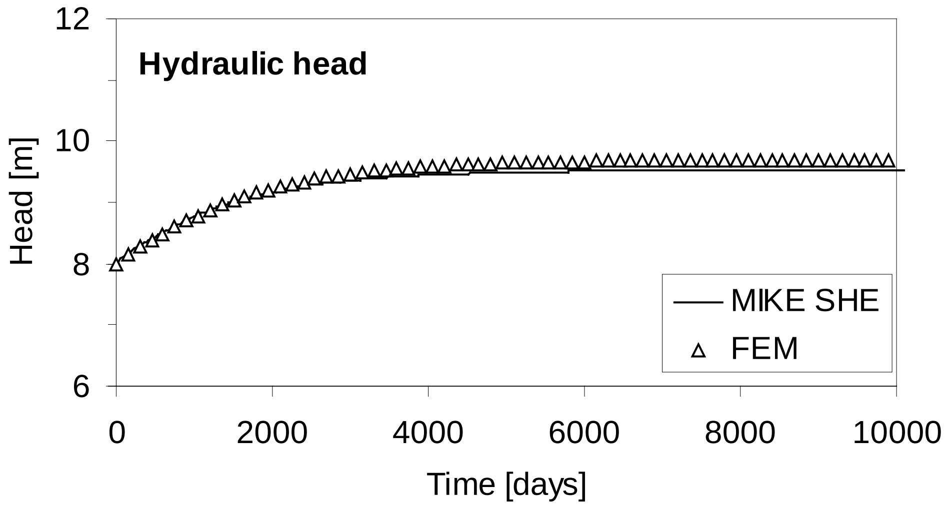


*Figure 4.6 Hydraulic head in (x, y) = (3000, 5000) (top element/layer) in a transient scenario.*

## 4.4 Real Test Case, Lille Å

The real test case, Lille Å, has only been carried out for a MIKE SHE setup. To set up the finite element model for a real test case requires a drainage function and a description of the interaction between surface water and groundwater, which have not been implemented. The results for the MIKE SHE simulation are not presented, since the scope was to validate the finite element model.

# 5 Applications and Perspectives

Before the finite element code can be applied to real test cases, like the proposed model for Lille Å, a drainage function and the coupling between surface and groundwater system must be developed. Drainage flow could be described as a withdrawal in some or all top elements, which could fairly easy be implemented. More sophisticated drainage options were originally beyond the scope of this project.

The perspectives for using finite element in groundwater and integrated models are promising, but as a first thing it needs to be coupled with other modules, e.g., overland flow or the unsaturated zone. Secondly, it requires a graphical user interface (GUI). The present version is basically a 'raw' finite element source code. E.g., one has to edit the input file in ascii-format, and the grid generator is not fully automated and incorporated to the finite element model. Thirdly, the post-processing tools are not completely developed yet, but the format is prepared to make use of the DHI MIKE Zero standard. For time series evaluation the post-processing tool is ready and tested.

The finite element model has been tested against analytical solutions and a finite difference code (MIKE SHE). Generally, the finite element code performed well and for the circular island test example even faster and more accurate than MIKE SHE did. This is mainly because a circular island can be described with less calculation points when a finite element grid is used. The finite element code has not yet been optimised in terms of CPU-time, so presumably it can be improved to run faster than now. The two other test examples showed small discrepancies compared to MIKE SHE, but the reason for these discrepancies has not been found. Finally, the real test case has not been carried out.

# 6 References

Beckers, J., Molson, J. W., Martin, P. J., and Frind, E. O. (2000). Watflow/3D Version 2.0. A three-dimensional groundwater flow model with modules for automated calibration and parameter sensitivity analysis – User guide, Centre for Research in Earth and Space Technology, University of Waterloo, Ontario, Canada.

Istok, J. (1989). Groundwater Modeling by the Finite Element Method, Water Resources Monograph 13, American Geophysical Union.

# Appendix A

When the CPU times are compared it is important to know some facts about the compiler, solver method, accuracy criteria etc. For the finite element model the simulation times are written in the output file and these times are shown in the tables. For MIKE SHE the simulation times are estimated based on the difference in time between the setup files and output files are stored. If other processes are running during a MIKE SHE simulation it may effect the computational time.

## Finite Element:

| | |
|---|---|
| Compiler: | AB-Soft, Unix-machine |
| Method: | DAE: Implicit BDF-method (transient)<br>DAE: Newton-Raphson method (steady-state)<br>LES: GMRES with preconditioning using incomplete Gauss elimination |
| Accuracy: | DAE, absolute tolerance = 10E-6<br>DAE, relative tolerance = 10E-6<br>LES, tolerance = 10E-6 |
| Time step: | 1 day |
| No. of time steps: | Steady-state: 1<br>Circular island and K-variation: 14600 days (40 years)<br>Phreatic surface: 1000 days |

## MIKE SHE:

| | |
|---|---|
| Compiler: | Fortran PowerStation 4.0, PC |
| Method: | Preconditioned Conjugate Gradient (PCG)<br>The PCG solver provides a steady-state option (sets all storage coefficients equal to zero) |
| Accuracy: | Head iteration stop criteria = 10E-4 m<br>Water balance stop criteria = 10E-5 |
| Time step: | 1 day |
| No. of time steps: | Steady-state: 1<br>Circular island and K-variation: 14600 days (40 years)<br>Phreatic surface: 1000 days |